## **Chapter 1 The Multi-product Newsvendor Problem: Review, Extensions, and Directions for Future Research**

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<span id="page-2-0"></span> $1 - \sum_{\mathbf{d} \in \mathcal{C}} \int_{\mathbb{R}^d} \left( \log \frac{1}{\mathbf{d} \cdot \mathbf{d}} \right) \mathbf{1}_{\mathbf{d} \in \mathbb{R}^d} \mathbf{1}_{\mathbf{d} \in \mathbb{R}^d} \mathbf{1}_{\mathbf{d} \in \mathbb{R}^d} \mathbf{1}_{\mathbf{d} \in \mathbb{R}^d}$  $\frac{1}{4} \int_{0}^{1} \frac{1}{2} \int_{0}^{1} \frac{1}{2} \, e^{-\frac{1}{2} \int_{0}^{1} \frac{1}{2} \int_{0}^{1$  $\mathbb{E}[\Pi(\begin{array}{cc} * & * \\ 1, & 2, \ldots, \end{array}] = \sum$ 

 $\mathbf{6}$  N. Turken et al. Turken et al.

$$
\frac{\partial}{\partial \theta} = \sum_{i=1}^{n} \frac{1}{\sqrt{2\pi i}} \left(1 - \frac{1}{\sqrt{2\pi i}}\right)
$$
\n
$$
* = F^{-1}\left(\frac{1}{1 - \frac{1}{\sqrt{2\pi i}}}\right), \qquad (1, 10)
$$
\n
$$
* = F^{-1}\left(\frac{1}{1 - \frac{1}{\sqrt{2\pi i}}}\right), \qquad (1.10)
$$

 $1 - \frac{\overline{N}}{n} + \frac{1}{n}$  or  $\overline{r}$  and  $\overline{r}$   $\overline{r}$  and  $\overline{r}$  $\lim_{\epsilon \to 0} g_{\epsilon} \leq \lim_{\epsilon \to 0} g_{\epsilon}$  terms of  $(1, 0)$ , we can find the expected net benefit the expected net benefit the expected net benefit the expected net benefit to  $\epsilon$  $\mathcal{L}_{\mathbf{a}}$  the matrix of  $\mathcal{L}_{\mathbf{a}}$  of  $\mathcal{L}_{\mathbf{a}}$  (*EBMU*) at assume that assume that  $\mathcal{L}_{\mathbf{a}}$  at assume that  $\mathcal{L}_{\mathbf{a}}$  at a summary  $\mathcal{L}_{\mathbf{a}}$  at a summary  $\mathcal{L}_{\mathbf{a}}$  at a summary  $\mathcal{L}_{\math$  $E = [- + ][1 - F(^{*})] - [- |F(^{*})$ . (1.14)  $\mathcal{L}_{\mathbf{a}}$  ,  $\mathcal{L}_{\mathbf{B}}^E$   $\mathcal{L}^2$  is an analogous to the lattice to the lattice to the set of process to the set of the se  $\lim_{n\to\infty} \frac{1}{n} \log \frac{1}{n} \log \frac{1}{n} = \frac{1}{n} \log \frac{1}{n} \log \frac{1}{n}$ [long](#page-34-0) [left](#page-34-0) [tails.](#page-34-0)  $\mathbf{A}$  and  $\mathbf{A}$  et al. [\(2004](#page-34-0)) developed the exact solution for uniformly for uniformly for uniformly  $\mathbf{A}$ distribution demand and presented a generic iterative method (FIM) when the second contract of  $\mathbb{C}$  $d_{\rm B}$  demand distribution is general. The author considered the total budget as the total budget a  $\int_{0}^{1}$   $\int_{0}^{\infty}$   $\int_{0}^{\infty}$   $\int_{0}^{\infty}$   $\int_{0}^{\infty}$  different from  $\int_{0}^{\infty}$  different from  $\int_{0}^{\infty}$  different from  $\int_{0}^{\infty}$  different from  $\int_{0}^{\infty}$  disposal feed feed feel feel feel feel feel fe  $\sum_{i=1}^n a_i$  is  $x = b_i$  as in given  $a_i$  is a busing is abundant, the problem could be problem could  $\mathbf{b} = \frac{1}{2} \mathbf{b} - \frac{$  $\frac{1}{\sqrt{2}}$  the Lagrangian-based approach to solve the problem. The value of  $\lambda$  is contained of  $\lambda$  is contained of  $\lambda$  is contained by  $\lambda$  is contained by  $\lambda$  is contained by  $\lambda$  is contained by  $\lambda$  is contained  $\sin^2 \theta$  is the also almost discusses the positive  $g_{\rm max}$  demand distributions. The formula formula formula formula for  $\lambda$  when the demand is uniformly is uniformly is uniformly in the demonstration of  $\lambda$  $d_{\rm{max}}$  distribution between and distribution between and distribution  $d_{\rm{max}}$ 

**Threshold 2:**

<span id="page-5-0"></span>
$$
G^{(2)}_G = \sum_{i=1}^{\infty} F^{(-1)} \left( \frac{-\left(\theta^- + 1\right) + \cdots}{- +} \right), \tag{1.1}
$$

$$
\theta = \frac{1}{\sqrt{2\pi}} \int_{0}^{\pi} \left(\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{
$$

$$
\begin{array}{c}\n\overline{1}_{\text{eff}}\overline{
$$

$$
C = 2 \quad \frac{(2)}{G} \leq \quad G \leq \quad \frac{(1)}{G} \quad \dots \quad \frac{(m+1)}{G} \quad \frac{1}{G} \quad \frac{1}{G
$$

$$
F(\t^*) = \frac{- (\theta + 1) + \cdot}{- + \cdot}.
$$
 (1.21)



$$
G_{1} = \sum_{i=1}^{\prime} F^{-1} \left( \frac{-\left(\theta + 1\right) + \cdots}{- + \cdots} \right) < G. \tag{1.22}
$$



 $1$   $\frac{\sqrt{2}}{2}$   $\frac{1}{2}$   $\frac{1}{2}$  $s$ similar distribution. The second case is when the demand for each international for each international for each  $\frac{1}{2}$ a uniform distribution. The first heuristic distribution all of the items  $\frac{1}{\sqrt{2}}$ have a similar cost structure and similar shaped demonstration and similar shaped demonstration and requires  $\frac{1}{2}$  $\sum_{i=1}^n a_i$  the mean and variance of each distribution and the cost data. The cost data  $\sum_{i=1}^n a_i$  $s^{\frac{1}{2}}$ second heuristic ( $\frac{2}{\pi}$ ) is optimal when the demand is uniformly distributed for each  $\frac{1}{\pi}$  $\mathbb{E}_{\mathbf{z}} = \mathbb{E}_{\mathbf{z}} \left[ \mathbf{z} \mathbf{z} \right]_{\mathbf{z}}$  is a modification of  $\mathbf{z}$  of  $\mathbf{z}$  account for general costs  $\mathbf{z}$  $\frac{1}{2}$ structures based on the form of  $(12)$ . The authors use computational experiments to  $\frac{1}{2}$  for all  $s \mapsto \frac{1}{2} \left( \frac{1}{2} \right) \left( \frac{1}{2} \right)$  is the most effective one, especially at  $\frac{1}{2}$  is the capacity.  $\mathcal{L}_{\mathcal{L}}$  and  $(200)$  developed a binary search method to obtain the obtain the obtain the obtain the obtain tion. The matrix of marginal benefit function as  $f(x) = ( - \frac{1}{x} + \frac{(x+1)F(x)}{1-x})$ ),  $w_{\rm obs} = ( \pm 1)$  is a non-decreasing function of , when  $\geq 0$  and its inverse is a  $\pm 1$ strictly increasing function of when 1 − ≤ ( ) *<* 0. The authors find that  $t_{\rm obs}$  the optimal solution to the same as the same as the same as the same as the unconstraint  $\theta_{\rm obs}$  $\delta$ <sup>p</sup>timal solution when the budget constraint is not binding and is not binding and is not binding and is less than the solution of  $\epsilon$  $u_{\rm{max}}$  solution  $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$  is binding. In the are are are are are are are are are as a single solution is binding. If the second  $\frac{1}{2}$ nonzero optimal solutions, the solutions, the should equal equal experiments. When  $\frac{1}{\sqrt{2}}$  the budget constraint is binding in  $\left(\begin{array}{cc} 1 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right)$  (∗∗) = ( )  $\int_{a}^{b}$  the matrix  $\int_{a}^{b}$  (∗∗) . For the  $\lambda_3$  showed that  $1 - \leq (**) < 0$  $\mathbb{R}^{(**)}$  can be found using a binary search between the algorithm between the algorithm  $\mathbb{R}^{(**)}$  $t_{\rm{max}}$  developed finds the solution to the solution to the unconstrained problem and assesses section to the unconstraint of the unconstraint of the solution of the solution of the solution of the solution of the solut where  $\frac{1}{2}$  of  $\frac{1}{2}$  by  $\frac{1}{2}$  budget constraint. If the original value  $\frac{1}{2}$  for  $\frac{1}{2}$  solution does not be  $\frac{1}{2}$  $n-1$ not satisfy the condition, a binary search procedure is applied. This algorithm can be contained. This algorithm can  $p_{\mu\nu}$  and  $p_{\mu\nu}$  and  $p_{\mu\nu}$  are  $p_{\mu\nu}$  in the MPNP under any general demand variable under any general demand  $p_{\mu\nu}$  $d_1$ distribution and it can also provide a good approximate solution under discrete solution under discrete solution under discrete approximate solution under discrete solution under discrete solution under discrete solu demand distributions.  $Z_{\rm max}$  and  $Z_{\rm max}$  [\(2010](#page-36-1)) studied the MPNP with  $Q_{\rm max}$  capacity constraint, where the theorem is  $Q_{\rm max}$  $p$  products can be outside to an external facility at a higher cost. The cost of a higher cost. The  $\alpha$  $\frac{1}{2}$  and  $\frac{1}{2}$  ( $\frac{1}{2}$ ) and nonzero lead time (



<span id="page-8-0"></span> $1 - \frac{1}{\lambda}$ ,  $1 - 0$  is  $\lambda$ ,  $\frac{1}{\lambda}$  is  $\lambda$ ,  $\frac{1}{\lambda}$  is  $\lambda$ ,  $\frac{1}{\lambda}$ denotes a worst case distribution function function of the demand. The optimal solution of the demand. The optimal solution  $\alpha$ can be found that the use of this distribution $f_{\rm esc}$  solve solution is justified value of additional information information  $f_{\rm esc}$  $(E \wedge E) = \sum_{i=1}^{n} C^{N}(\bigcirc_{i=1}^{n} - \sum_{i=1}^{n} C^{N}(\bigcirc_{i=1}^{n} - \bigcirc_{i=1}^{n} - \bigcirc_{i=1}^{n}$  $\mathbf{b}$ er studied in the paper.  $\lceil \cdot \rceil$  and  $\lceil \cdot \rceil$  and  $\lceil \cdot \rceil$  [\(2006](#page-35-0))

 $\sum_{i=1}^{\infty}$  order to find the single theory find the single the single that satisfy  $\sum_{i=1}^{\infty}$  [\(1.24\)](#page-8-0) for the single single the single the single singl  $I_{\alpha} = \frac{1}{2} \int \frac{1}{2} \, dx$  individual and global and global and global and global  $\alpha$ values are the same. Otherwise, if *P*<sup>∗</sup> *T* **P**  $\frac{a}{b}$  **P**  $\frac{a}{b}$  **P**  $\frac{a}{b}$  **P**  $\frac{b}{b}$  **F**  $\frac{b}{b}$  **P**  $\frac{b}{c}$  **b**  $\frac{b}{c}$  **P**  $\frac{c}{c}$  **b**  $\frac{c}{c}$  **b**  $\frac{c}{c}$  **c**  $\frac{c}{c}$  **c**  $\frac{c}{c}$  **b**  $\frac{c}{c}$  **c**  $\frac{c}{c}$  **b**  $\frac{c}{c}$  **c**  $\$  $T = \frac{1}{2} \int_{0}^{1} \frac{1}{\sqrt{2}} \int_{0}^{\frac{\pi}{2}} \frac{1}{\sqrt{2}} e^{-\frac{1}{2} \left( \frac{\pi}{2} \right)^2} \int_{0}$  $T_{\rm eff}$  and also applied the authors applied the species  $T_{\rm eff}$  and  $T_{\rm eff}$  as well. While  $T_{\rm eff}$  $d_{\rm eff}$  in a mathematical expression for  $d_{\rm eff}$  and  $d_{\rm eff}$  introduced two introduced two introduced two introduced two introduced two intervals of  $d_{\rm eff}$  introduced two intervals of  $d_{\rm eff}$  intervals of  $d_{\rm eff}$  i  $d_{\text{max}} = \frac{1}{2} \int_{\mathcal{X}} \left[ \frac{1}{2} \int_{\mathcal{X}} \frac{d\mu}{d\mu} \right] \left[ \frac{1}{2} \int_{\mathcal{X}} \frac{d\mu}{d\mu} \$  $p \rightarrow p$   $p \rightarrow q$   $\in \mathbb{R}^d$ ,  $p \rightarrow q$   $\infty$   $p \rightarrow q$   $q \rightarrow q$  $a_n$   $a_n$   $a_{n+1}$   $a_{n+2}$   $a_{n+1}$   $a_{n+2}$   $a_{n+1}$  $\mathbf{z}_t^{(t)}$  $\frac{1}{\epsilon_1}$   $0 \leq 1 \leq 1$   $\epsilon_2$   $1 = (\epsilon_1 + 1 + 2)$ 



1 The Multi-product Newsvendor Problem*...* 15 Consistent with the previous multidimensional newsvendor models, the newsvendor networks are defined by a linear production technology, which describes how inputs (supply) is transformed into outputs of fill end–product demand, a linear financial structure, and a probability distribution of end–product

demand at the junction  $\theta$  is the solution of  $\theta$  in the nondiscount price. If the nondemand for the product at the product at the non-then discounting the price is the price is then discounting the price is the pric  $\lambda = 0$  the product results in a proportional demand demand. Observations on  $\theta = 0$  $t_{\rm obs} = \frac{1}{2\pi} \int_{\gamma_{\rm obs}}^{\gamma_{\rm obs}} \frac{1}{\gamma_{\rm obs}} \frac{d\gamma_{\rm obs}}{d\gamma_{\rm obs}} \frac{1}{\gamma_{\rm obs}}$  $i_n$  and  $j_n$  and  $j_n$  and  $j_n$  in an  $(i_n, 0, \ldots, 1)$  in a satisfying demand  $(i_n, 0, \ldots, 1)$ and order  $\alpha$  and  $\alpha$  and  $\alpha$  all products, when compared to the corresponding levels in the corresponding  $f(x) = \frac{1}{2\pi} \frac{1}{2\pi}$  $m_{\rm s}$  multiple and single discounts indicate that using  $m_{\rm s}$  in discounts in solutions in stead of  $m_{\rm s}$  $\alpha$  in discounting to the salvage value of salvage value  $\alpha$  of  $\alpha$  $\overline{z}$ ,  $l_{\rm T}$ :  $S_{\text{max}}$   $\sum_{i=1}^{\infty}$  [\(2010\)](#page-35-1),  $S_{\text{max}}$  [\(2011](#page-35-2))  $\sum_{i=1}^{\infty}$  [\(2010](#page-36-2)) in vestigated the MPNP with supplier  $\frac{1}{2}$  and a budget constraint constraint and a budget constraint, and the effect of the eff  $f_{\rm eff}$  (  $f_{\rm eff}$  )  $f_{\rm eff}$  (  $f_{\rm eff}$  ) order  $f_{\rm eff}$  and  $f_{\rm eff}$  presented a mixed integer presented a mixed i  $\overline{a}$  in  $\overline{b}$  programming  $\overline{a}$  is formulate to formulate the problem. The problem is problem. relaxation approach is demonstrated by means of numerical tests. Finally, the means of  $\frac{1}{\sqrt{2}}$  $\tilde{\rho}$  and the multiple constraints including space or other resources or other resources or other resources  $\tilde{\rho}$ limitations. It is assumed that suppliers provide a large suppliers provide a large suppliers of  $\frac{1}{2}$  and  $\frac{1}{2}$  an newsversvendor faces uncertain demand for  $\frac{1}{2}$  $d_{\rm B}$ density function for each product is assumed to be given. To solve the problem, the problem, the authors use the problem, the authors use the  $\frac{1}{2}$ methods to find upper and upper and lower bounds, as well as well as  $\frac{1}{2}$ They related the budget constraint (instead of discount constraints that potentially  $\frac{1}{2}$  $d_{\mathbf{a}}$  ,  $\int_{\mathbf{b}} d\mathbf{x}$  ,  $\mathbf{c}$  and  $\mathbf{c}$  as it results in a classical newsventility in a classical newsventility in a classical newsventility in  $\mathbf{c}$  $\frac{1}{2}$  with discount constraints. The computational results in discount that the algorithm is algorithm in discount of the algorithm is a substitution of the algorithm is algorithm in discount of the algorithm is algor extremely effective for the newsvendor model with supplier  $\frac{1}{2}$  and  $\frac{1}{2}$  and  $\frac{1}{2}$  and  $\frac{1}{2}$ a budget constraint (in terms of both solution  $\frac{1}{2}$  budget computing times). The solution of the solution of  $\frac{1}{2}$  $c_0$  indicate  $c_1$  results for the multi-constraint case also indicate that the proposed  $c_1$  $\alpha_1 \alpha_2 \alpha_3 \alpha_4 \alpha_5 \alpha_7 \alpha_8 \alpha_9$  with multiple constraints.  $I_{\text{max}}$  (a different extension, C<sub>hen</sub> and C<sub>hen</sub>  $\alpha$   $\beta$  and  $\beta$ newsvendor model under a budget constraint with the addition of a reservation  $\delta$   $\frac{1}{2}$ ,  $\frac{1}{2}$  and  $\frac{1}{2}$  is  $\frac{1}{2}$  and  $\frac{1}{2}$  in the demand uncertainty of  $\frac{1}{2}$  $p_{\rm max}$  products. Under the reservation policy studies in the reservation policy studies in the rate is of the rate is offered in the rate in the rate is of the rate is offered in the rate is of the rate is offered in th  $\tau$  to consumers in order to induce the make a reservation and buy in advance. The authors propose a general algorithm, namely the MCR algorithm, namely the MCR algorithm, which finds  $\frac{1}{2}$  $\frac{1}{2}$  of  $\frac{1}{2}$  order  $\frac{1}{2}$  order  $\frac{1}{2}$  order  $\frac{1}{2}$  order to maximize the total total total the total tota  $e^{\frac{1}{2}(\theta-\theta)}$ expected profit under the budget constraints the effective the efficiency of efficiency of efficiency of efficiency of  $\theta$  $\frac{1}{2}$  or posed algorithm,  $\frac{1}{2}$  and compare they solve a numerical example and compare the compare theorem is  $\frac{1}{2}$  $c_{\rm{max}}$  multi-product budget-constraint newsvendor model (CMC model)  $\int_{0}^{\infty}$  $t_{\rm eff}$  multi-product budget-constraint newsvendor model with the reservation policy.  $N_{\rm tot}$  results show that the total expected profit obtained from the MCR is  $g_{\rm max} = \frac{1}{\pi} \int_{0}^{\pi} f(x) \, dx$  that  $\int_{0}^{\pi} f(x) \, dx$  is times policy proposed in the reservation policy proposed model. The difference between the profits of the profits of the profits of the value of the value of the value  $\lambda$ of information. Thus, we can conclude the decision to a decision to a reservation of the reservati  $p \leftrightarrow p$  and  $p \leftrightarrow p$  in the trade-off between the information value and the information value and the cost including  $p \leftrightarrow q$ to establish the willingness function and extra-demand functions.



1 
$$
\sum_{i=1}^{n} \frac{1}{i} \int_{0}^{1} \frac{e^{-\frac{1}{2}t^{2}}}{t^{2}} e^{-\frac{1}{2}t^{2}} e
$$

22  
\n
$$
u + \vec{a} = -3
$$
  
\n $u + \vec{a} = -3$   
\n $u + \vec{a} = 3$   
\n $u + \vec{a} = 4$   
\n $u + \vec{a} = 4$ 

## *1.3.1 One-Way Substitution*

$$
\frac{1}{2} \int_{0}^{2\pi} \frac{1}{2} e^{-2x} \int_{0}
$$

$$
G(\overrightarrow{\hspace{0.5cm}}\phantom{+}, \overrightarrow{\hspace{0.5cm}}\phantom{+}) = \sum_{\rule{0pt}{3mm} i \rightarrow \gamma}^{\ell} \sum_{j=1}^{\mathsf{N}} \sum_{l=1}^{\mathsf{N}} \frac{1}{l} \sum_{j=1}^{\mathsf{N}} \frac{1}{l} \sum_{j=1}^{\mathsf{N}} \frac{1}{l} \sum_{l=1}^{\mathsf{N}} \frac{1}{l} \sum_{l=1}^{\mathsf{N
$$

 $E_{\text{tot}}$ 

 $\bullet$ <br/> $\bullet$ 

$$
1 + \sum_{i=1}^{\infty} = 1, \ldots, N, \quad + \sum_{i=1}^{N} = 1, \ldots, N. \tag{1.32}
$$



 $\mathbf{t} \in \mathbb{R}$  and  $\mathbf{t} \in \mathbb{R}$  and  $\mathbf{t} \in \left(2010\right)$  $\mathbf{t} \in \left(2010\right)$  $\mathbf{t} \in \left(2010\right)$ 

1  $\frac{1}{\epsilon}$   $\frac{1}{\epsilon}$   $\frac{1}{\epsilon}$   $\epsilon$   $\frac{1}{\epsilon}$   $\frac{1$ 

+ <sup>1</sup> 0 <sup>∞</sup> 2 ( <sup>1</sup> − <sup>1</sup>) ( <sup>1</sup>*,* <sup>2</sup>)d 2d <sup>1</sup> − <sup>1</sup> <sup>1</sup> − <sup>2</sup> <sup>2</sup> + <sup>1</sup> <sup>1</sup> 0 <sup>2</sup> 0 ( <sup>1</sup> − <sup>1</sup>) ( <sup>1</sup>*,* <sup>2</sup>)d 2d <sup>1</sup> + <sup>1</sup><sup>+</sup> <sup>2</sup><sup>−</sup> <sup>2</sup> 0 <sup>1</sup><sup>+</sup> <sup>2</sup> 2 ( <sup>1</sup> + <sup>2</sup> − <sup>1</sup> − <sup>2</sup>) ( <sup>1</sup>*,* <sup>2</sup>)d 2d <sup>1</sup> + <sup>2</sup> <sup>2</sup> 0 ( <sup>2</sup> − <sup>2</sup>) <sup>2</sup>( <sup>2</sup>)d <sup>2</sup> − <sup>1</sup> <sup>∞</sup> 1 ( <sup>1</sup> − <sup>1</sup>) <sup>1</sup>( <sup>1</sup>)d <sup>1</sup> − <sup>2</sup> <sup>∞</sup> 1 <sup>∞</sup> 2 ( <sup>2</sup> − <sup>2</sup>) <sup>1</sup>( <sup>1</sup>*,* <sup>2</sup>)d 2d <sup>1</sup> + <sup>1</sup> 0 <sup>∞</sup> 1+ 2− 1 ( <sup>1</sup> + <sup>2</sup> − <sup>1</sup> − <sup>2</sup>) ( <sup>1</sup>*,* <sup>2</sup>)d 2d <sup>1</sup> *.* (1.35)

[Cai](#page-34-6) [et](#page-34-6) [al.](#page-34-6) [\(2004](#page-34-6)) used a similar expected profit function as above and proved that it is concave and submodular. Using this property, the optimal order quantities can be found by setting the derivatives with respect to <sup>1</sup> and <sup>2</sup> equal to zero. If we define *G*( <sup>1</sup>*,* <sup>2</sup>) = <sup>1</sup> 0 <sup>1</sup><sup>+</sup> <sup>2</sup><sup>−</sup> <sup>1</sup> <sup>0</sup> ( <sup>1</sup>*,* <sup>2</sup>)d 2d 1, the following holds:

$$
F_1\left(\begin{array}{c} * \\ 1 \end{array}\right) + \frac{\left(\begin{array}{c} 2+2 \end{array}\right) - 1}{\left(\begin{array}{c} 1+1 \end{array}\right) - \left(\begin{array}{c} 2+2 \end{array}\right)} G\left(\begin{array}{c} * \\ 1 \end{array}\right) = \frac{\left(\begin{array}{c} 1+1 \end{array}\right) - 1}{\left(\begin{array}{c} 1+1 \end{array}\right) - \left(\begin{array}{c} 2+2 \end{array}\right)},\tag{1.36}
$$

$$
F_2\left(\begin{array}{c} z \\ 2 \end{array}\right) + \frac{\left(\begin{array}{cc} 2+2 \end{array}\right) - 1}{\left(\begin{array}{cc} 2+2 \end{array}\right) - 2} \left[ G\left(\begin{array}{cc} * & * \\ 1 & 2 \end{array}\right) - F\left(\begin{array}{cc} * & * \\ 1 & 2 \end{array}\right) \right] = \frac{\left(\begin{array}{cc} 2+2 \end{array}\right) - 1}{\left(\begin{array}{cc} 2+2 \end{array}\right) - 2}. \quad (1.3)
$$

*F* ( <sup>∗</sup>) represents the probability of all of the demand for item being sat-  $\int_{\mathcal{A}}$  **i c** is the stock level is  $\int_{\mathcal{A}}^{x}$  **i** and  $\int_{\mathcal{A}}^{x}$  **i**  $\binom{*}{1}$ ,  $\binom{*}{2}$  =  $\int_0^1$  <sup>2</sup>  $\int_{a}^{1} \int_{0}^{2} (1, 2) \int_{0}^{1} (1, 2) \int$ \*<br>1<sup>*,*</sup> 2  $(\frac{2}{2})^+$ *F*( $\frac{1}{1}$ <sup>1</sup>*,* <sup>∗</sup>  $C_{\alpha}$  ,  $\alpha$  between property  $\alpha$  of the optimal order  $\alpha$  or  $\alpha$  the optimal order  $\alpha$   $\alpha$   $\alpha$   $\beta$  $s_{1} = \frac{1}{2} \int_{0}^{1} 0.4 \int_{0}^{\infty} x \left( \int_{0}^{\infty} \frac{1}{2} e^{i \frac{3}{2} x} \right) e^{i \frac{3}{2} x} \left( \int_{0}^{\infty} e^{i \frac{3}{2} x} \right) e^{i \frac{3}{2} x} \left( \int_{0}^{\infty} e^{i \frac{3}{2} x} \right) e^{i \frac{3}{2} x} \left( \int_{0}^{\infty} e^{i \frac{3}{2} x} \right) e^{i \frac{3}{2} x} \left( \int_{0}^{\infty} e^{i \frac$  $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$  $C_1$  decreases in price of item increases in price of item 1 decreases the optimal quantity for  $\frac{1}{2}$ .  $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 &$  $t_{\rm c}$  the optimal order  $\frac{1}{2}$  order  $\frac{1}{2}$  related to the individual to the individua  $d_{\rm th}$  states that the variance of item after the variance of item affects the optimal quantity of  $\sigma$  $\frac{1}{2}$  item reversely. In this paper, the authors showed that the expected profits and the ex fill rate can be improved by using substitution.

<span id="page-23-0"></span>

### *1.3.2 Two-Way Substitution*

 $U_{\rm{max}}$  the items case, in two-way substitution, in the interval case, each of the interval case, each of the interval case, in the  $\mathbf{J}_{\text{total}}$  to  $90\text{y}$  and the demand for another one. This one. This one. This one. The demand the demand the demand the demand of  $\mathbf{J}_{\text{total}}$  $f(x)$  is the item is higher than the quantity ordered and the substitute ordered and the substitute of the substi item is lower than the quantity of  $\mathbb{R}$  ordered. McGilliver [\(1978](#page-35-3)) and Parlar and Parlamentary and Parlamentary and Parlar and Pa and  $\alpha$  [\(1984\)](#page-35-4) assumed whenever substitution is possible, there is a probability is a probability of  $\alpha$  $t_{\rm eff}$  a customer will accept a substitute product. In Parlar's case, this product product  $p_{\rm eff}$ was between  $0$  and  $1$ , whereas it was fixed for  $\mathbb{X}$  whereas  $\mathbb{Y}$  was  $\mathbb{Y}$  and  $\mathbb{Y}$  an the demand, , is assumed to be normally distributed with a mean of and a  $s_{\text{total}}$   $\left\{ \begin{array}{ccc} \n\mu & \rightarrow & \sigma = \beta \sigma \n\end{array} \right\}$   $\sigma_{\text{total}}$   $\sigma_{\text{total}}$   $\left\{ \begin{array}{ccc} \n\mu & \mu & \mu \n\end{array} \right\}$   $\sigma_{\text{total}}$   $\sigma_{\text{total}}$   $\sigma_{\text{total}}$  $Q_{\alpha}$  **b**<sub> $\alpha$ </sub>  $I_{\alpha}$  **F**<sub> $\alpha$ </sub>  $I_{\alpha}$  **F**<sub> $\alpha$ </sub><sup> $\beta$ </sup>*<sub>* $\beta$ *</sub>*  $\gamma$  $C = \sigma G_{\alpha}$  (*K*). variable costs and shortage costs and shortage costs are also assumed to be a substitution of the sub identical. This assumption is just that in reality when the fact that is just that in reality when two items are  $\frac{1}{2}$ وتوسع تلابي والمتوجب وتصادمها وسائط وحطته وتعاطي المستعمل المساحي المستعمل والمحددة considered in the paper. The notation used in their paper is shown on Table [1.1.](#page-23-0)  $\mathcal{G}_{\mu}$   $\mathcal{$ the partial derivative of  $\mathbf{R}$  **C**, we find  $\mathbf{R}$  **P**<sub>≥</sub>( $\mathbf{R}$  ∗) =  $\frac{\mathbf{R}}{k}$  $f(x) = 1, \ldots, N$  is  $\mathbb{Z}_{p}$ ,  $\mathbb{Z}_{p}$  is equal property  $\begin{pmatrix} 0 & 0 \\ 0 & x \end{pmatrix}$ ,  $\begin{pmatrix} 0 \\ 0 \end{pmatrix} = 0$ ,  $\geq \begin{pmatrix} 0 \\ 0 \end{pmatrix} + G_{p}(0)$ , ETRC can be reduced to:

$$
\boldsymbol{\overline{E}}^{\bullet} \mathbf{X} \mathbf{C}(\mathbf{K}_{1}^{*}, \mathbf{K}_{2}^{*}, \ldots, \mathbf{K}^{*}) = \frac{1}{2} \mathbf{A}^{\bullet} \mathbf{A}^{\bullet} + \sigma_{1} \mathbf{A}(\mathbf{K}^{*}) \sum_{i=1}^{N} \boldsymbol{\beta}. \qquad (1.3)
$$

If we assume that the third is full demand that there is full demand to the perfect of  $\mathcal{L}$  $s_{\lambda}$   $\mu$ <sup>1</sup>,  $\mu$ <sup>2</sup>,  $\sigma$   $\mu$ <sub>2</sub>,  $\tau$   $\sigma$   $\mu$ <sub>2</sub>,  $\mu$ <sub>2</sub>,  $\mu$ <sup>3</sup>,  $\mu$ <sub>2</sub>,  $\mu$ <sub>2</sub>,  $\sigma$ <sub>2</sub>, all items is smaller than the total stock up-to level. The total shortage and on-hand inventory decrease the same amount by the transfer sales; therefore, the total net



1 
$$
\sum_{i=1}^{10} \frac{1}{i} \int_{0}^{1} \frac{1}{1} \int_{0}^{1} \
$$

and based on this, the expected profit function is: *E*[π( <sup>1</sup>*,* <sup>2</sup>)] = *E* [ <sup>1</sup>*Min*( <sup>1</sup>*,* <sup>1</sup>) + <sup>2</sup>*Min*[ <sup>2</sup>*,* <sup>2</sup> + ( <sup>1</sup> − <sup>1</sup>) +]− <sup>1</sup> <sup>1</sup> − <sup>2</sup> <sup>2</sup> + <sup>1</sup>[ <sup>1</sup> − <sup>1</sup> −( <sup>2</sup> − <sup>2</sup>) +] <sup>+</sup> + <sup>2</sup>( <sup>2</sup> − <sup>2</sup>) <sup>+</sup> − <sup>1</sup>( <sup>1</sup> − <sup>1</sup>) + − <sup>2</sup>[ <sup>2</sup> − <sup>2</sup> −( <sup>1</sup> − <sup>1</sup>) +] + = <sup>1</sup> <sup>1</sup> 0 <sup>1</sup> <sup>1</sup>( <sup>1</sup>)d <sup>1</sup> + <sup>∞</sup> 1 <sup>1</sup> <sup>1</sup>( <sup>1</sup>)d <sup>1</sup> + <sup>1</sup> 0 <sup>∞</sup> 2 ( <sup>2</sup> − <sup>2</sup>) ( <sup>1</sup>*,* <sup>2</sup>)d 2d <sup>1</sup> + <sup>2</sup> <sup>2</sup> 0 <sup>2</sup> <sup>2</sup>( <sup>2</sup>)d <sup>2</sup> + <sup>∞</sup> 2 <sup>2</sup> <sup>2</sup>( <sup>2</sup>)d <sup>2</sup> + <sup>1</sup> 0 <sup>∞</sup> 2 ( <sup>1</sup> − <sup>1</sup>) ( <sup>1</sup>*,* <sup>2</sup>)d 2d <sup>1</sup> − <sup>1</sup> <sup>1</sup> − <sup>2</sup> <sup>2</sup> + <sup>1</sup> <sup>1</sup> 0 <sup>2</sup> 0 ( <sup>1</sup> − <sup>1</sup>) ( <sup>1</sup>*,* <sup>2</sup>)d 1d <sup>2</sup> + <sup>1</sup><sup>+</sup> <sup>2</sup><sup>−</sup> <sup>2</sup> 0 <sup>1</sup><sup>+</sup> <sup>2</sup> 2 ( <sup>1</sup> + <sup>2</sup> − <sup>1</sup> − <sup>2</sup>) ( <sup>1</sup>*,* <sup>2</sup>)d 1d <sup>2</sup> + <sup>2</sup> <sup>1</sup> 0 <sup>2</sup> 0 ( <sup>2</sup> − <sup>2</sup>) ( <sup>1</sup>*,* <sup>2</sup>)d 2d <sup>1</sup>

$$
t \stackrel{\bullet}{\bullet} \stack
$$





$$
e^{-z} t^{\frac{1}{2}} \longrightarrow e^{-z} t^{\frac{1}{2}} = \frac{2}{2(\frac{1}{1}2^{-2})},
$$
\n
$$
2(\frac{1}{2}, \frac{1}{2}) = \frac{1}{2(\frac{1}{1}2^{-2})},
$$
\n
$$
(1.60)
$$
\n
$$
2(\frac{1}{2}, \frac{1}{2}) = \frac{1}{2(\frac{1}{1}2^{-2})},
$$
\n
$$
(1.61)
$$

$$
\begin{array}{ccccccccc}\n\bullet & 1 & = & (1 + 1) - \int_{1}^{1} (\varepsilon_{1} - 1) & (\varepsilon_{1}) & \varepsilon_{1} - 2 + \mu_{1} & \varepsilon_{1} & 2 = & (2 + 2) \\
-\int_{2}^{2} (\varepsilon_{2} - 1) & (\varepsilon_{2}) & \varepsilon_{2} - 1 + \mu_{2} & \varepsilon_{2} & \varepsilon_{2} & \varepsilon_{2} & \varepsilon_{2}\n\end{array}
$$

1  $\frac{\pi}{e} \int_{0}^{1} e^{i\pi} e^{-i\pi} e^{i\pi}$  Multi-product  $\frac{35}{2}$ 

$$
- (+ - +1)
$$
  
= (- ) (+ - - ) - (-  $\int$  + (- ) $\mu$   
-(- - + ) $\int$  ( $\varepsilon$ -<sup>1</sup>) ( $\varepsilon$ <sub>•</sub>  $\varepsilon$ , (1.63)

 $\mathbf{z} = -(- + - \cdot)$ .  $\mathbf{z} = \mathbf{z}[\Pi(\mathbf{z}_1, \mathbf{z}_2, \mathbf{z}_1, \mathbf{z}_2)] = \mathbf{E}[\Pi_1]$  $+$  *E*[ $\Pi_2$ ]  $\mathcal{L}_{\mathcal{L}}$ 

$$
\frac{\partial \mathbf{E}[\Pi]}{\partial} = ( - ) + ( - ) + ( - + )[1 - F(\mathbf{E})](1 - ) +
$$

$$
- \{ ( - ) + ( - + )[1 - F(\mathbf{E})] \}.
$$
(1.64)

$$
\frac{\partial^2 \mathbb{E}[H]}{\partial^2} = -(- - + ) (1 - )^2 \quad ( ) - (- - + )^2 \quad ( ). \tag{1.65}
$$

$$
\frac{\partial^2 \mathbb{E}[II]}{\partial_1 \partial_2} = -(\mathbf{1} - \mathbf{1} + \mathbf{1})(1 - \mathbf{1}) \quad (1 - \mathbf{1}) \quad (1 - \mathbf{1} - \mathbf{1}) \quad (1 - \mathbf
$$

$$
\bullet \bullet \bullet \bullet = ( \quad -
$$

# 1  $\sum_{i=0}^{\infty}$   $\int_{0}^{1}$  or  $\sum_{i=1}^{\infty}$  Multi-product Newsversuendor Problema 27.

### **References**

<span id="page-34-7"></span><span id="page-34-6"></span><span id="page-34-5"></span><span id="page-34-4"></span><span id="page-34-3"></span><span id="page-34-2"></span><span id="page-34-1"></span><span id="page-34-0"></span> $\mathcal{A}^{\mathbf{L}}$  ,  $\mathcal{A}^{\mathbf{R}}$  ,  $\mathcal{A}^{\mathbf{R}}$  are expansion approach to the multi-set of multi-set product newsvendor problem with side constraints. *European Journal of Operational Research*, *1*<sub>6</sub>(3), 160<sup>-1</sup><sub>1</sub>.  $A^{\mathbf{R}}$  and  $A^{\mathbf{R}}$  and  $B^{\mathbf{R}}$  and  $B^{\mathbf{R}}$ .  $\mathbf{R}$  (2005a). An analysis of the multi-problems problems problems problems both problems  $A$  $\frac{1}{\sqrt{2}}$  a budget constraint. International *I*<sub>c</sub>  $\frac{1}{\sqrt{2}}$  *Production E*  $\frac{1}{\sqrt{2}}$  (3), 2, 6 30.  $A_{\alpha}$ ,  $A_{\alpha}$ ,  $B_{\alpha}$ ,  $B_{\alpha}$ ,  $B_{\alpha}$ ,  $C_{\alpha}$ ,  $C_{\alpha$  $\zeta_{\mathbf{r}}$  constraints.  $\zeta_{\mathbf{r}}$  and  $\zeta_{\mathbf{r}}$  and  $\zeta_{\mathbf{r}}$  **and**  $\zeta_{\mathbf{r}}$   $A_{\alpha}$   $\mathcal{R}_{\alpha}$  ,  $\mathcal{R}_{\alpha}$  is  $\mathcal{R}_{\alpha}$ ,  $\mathcal{R}_{\alpha}$  and  $\mathcal{R}_{\alpha}$  and  $\mathcal{R}_{\alpha}$  iterative indicates in the generic iterative indicates in the generic iterative indicates in the generic iterative indicates in  $m<sup>2</sup>$  and  $m<sup>1</sup>$  -  $n<sup>1</sup>$ 

#### 3  $\sim$  1. Turken et al. T

<span id="page-35-4"></span><span id="page-35-3"></span><span id="page-35-2"></span><span id="page-35-1"></span><span id="page-35-0"></span> $L_{\alpha\alpha}$ ,  $R_{\alpha\beta}$ ,  $L_{\alpha\beta}$ ,  $\frac{1}{2}$  **b**<sub>2</sub> **d**<sub>2</sub> **c**<sub>1</sub>, *1*(2), 3, 2<sub>4</sub>0<sub>1</sub>.  $L_{\alpha\alpha}$ ,  $\alpha$ ,  $\alpha$ ,  $(L_{\alpha\alpha}$ ,  $(L_{\alpha\alpha}$ ,  $L_{\alpha\alpha}$ ,  $L_{\alpha\alpha}$ , product multi-constraint newsbox problems, applications, applications, applications, applications, applications, applications, applications, applications, formulation and solution. *Journal of Operations Management*, *13*(2), 153–162.  $\mathcal{L}_{\mathcal{L}}$ ,  $\mathcal{L}_{\mathcal{L}}$ ,  $\mathcal{L}_{\mathcal{L}}$  and  $\mathcal{L}_{\mathcal{L}}$  can capacitated multiple-perioduct single-perioduct single-perioduct single-perioduct single-perioduct single-perioduct single-perioduct single-perioduct s inventory problem. *European Journal of Operational Research*, *94*(1), 29–42.  $L_{\rm max}$  ,  $L_{\rm max}$ ,  $L_{\rm max}$  (1997). Some results on integrating a multi-item multi-constraint single-constraint single-constraint single-constraint single-constraint single-constraint single-constraint single-constraint  $p_{\rm e}$   $\frac{1}{2}$   $\$  $M_{\rm g}$  ,  $\kappa$  ,  $\kappa$   $\kappa$ substitution. *Operations Research*, *49*(3), 334–351.  $M_{\rm tot}$  (1,  $\frac{1}{\alpha}$  ,  $\frac{1}{\alpha}$ ,  $\frac{1}{\alpha}$ ,  $\frac{1}{\alpha}$ ,  $\frac{1}{\alpha}$ ,  $\frac{1}{\alpha}$ ,  $\frac{1}{\alpha}$ ,  $\frac{1}{\alpha}$ ,  $\frac{1}{\alpha}$ ,  $\frac{1}{\alpha}$ ,  $\frac{1}{\alpha}$ ,  $\frac{1}{\alpha}$ ,  $\frac{1}{\alpha}$ ,  $\frac{1}{\alpha}$ ,  $\frac{1}{\alpha}$ ,  $\frac{1}{\alpha}$ ,  $\frac{1}{\alpha}$ ,  $\frac$  $d_{\mathbf{a}} = \mathbb{E} \cdot \mathbf{a} \cdot \mathbf{A} \cdot \mathbf{F} \cdot \mathbf{O} \cdot \mathbf{A}$ , (1), 4-63.  $\mathcal{M}_{\text{max}}$ , J.,  $\mathcal{R}_{\text{max}}$ ,  $\mathcal{M}_{\text{max}}$ ,  $(2002)$ . Newsversion in the investor management and capacity management and investment with discretionary activities. *Manuacturing Service Operations Management*, (4), 313–335. Moon, I., & Silver, E. A. (2000). The multi-item newsvendor problem with a budget constraint and fixed ordering costs.

<span id="page-36-2"></span><span id="page-36-1"></span><span id="page-36-0"></span>1  $\frac{1}{\epsilon}$   $\frac{1}{\epsilon}$  or  $\frac{1}{\epsilon}$  and  $\frac{1}{\epsilon}$  is the  $\frac{1}{\epsilon}$  such that  $\frac{1}{\epsilon}$  3  $\frac{1}{2}$  is a budget constraint multi-item newsborough multi-item newsborough a budget constraint. **International**  $\mathbf{a} = \begin{bmatrix} \mathbf{b} & \mathbf{c} & \mathbf{c} \\ \mathbf{c} & \mathbf{d} & \mathbf{c} \end{bmatrix}$  **6** (3), 213–226.  $Z_{\text{max}}$ ,  $\mathcal{R}$ ,  $\mathcal{Z}$ . (2010). Multi-product newsbox problem with limited capacity and outsource- $\mathcal{O}^{\mathfrak{t}}$  *C*  $\mathcal{O}^{\mathfrak{t}}$  *C*  $\mathcal{O}^{\mathfrak{t}}$  *R*  $\mathcal{O}^{\mathfrak{t}}$  *R*  $\mathcal{O}^{\mathfrak{t}}$  *(Reserve)*, *202*(1), 10 113.  $\frac{1}{2}$  p.,  $\frac{1}{2}$ ,  $\frac{2010}{2}$ .  $0 \approx 1$ ,  $00 \approx 1$ ,  $0 \approx 1$ ,  $100 \approx 1$ ,  $0 \approx 1$ ,  $0 \approx 1$ constraint. *Computers & Industrial Engineering*, *58*, 759–765.  $\frac{1}{2}$ ,  $\frac{1}{2}$ ,