

Chapter 1

The Multi-product Newsvendor Problem: Review, Extensions, and Directions for Future Research

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$(i = 1, \dots, N)$

(\otimes)

N

10.100 / -1-4614-3600-3-1, 2012

3

$$\begin{aligned}
 & 1 \quad \int_{t_0}^t \dots \int_{t_0}^{t_1} \dots \int_{t_0}^{t_2} \dots \int_{t_0}^{t_{n-1}} \dots \\
 & \int_{t_0}^t \dots \int_{t_0}^{t_1} \dots \int_{t_0}^{t_2} \dots \int_{t_0}^{t_{n-1}} \dots \\
 & \mathbb{E}[\Pi(\cdot_1, \cdot_2, \dots, \cdot_n)] = \sum
 \end{aligned}$$

$$\frac{\partial}{\partial} = \sum_{=1} - . \quad (1,)$$

• (1.) • • t • 0, • o t • z • f • t • t • • o • t • * t z

$$* = F^{-1} \left(\frac{- - \lambda +}{- +} \right), \quad (1.10)$$

• • $\lambda \geq 0.$

1 $\nabla_t \{ - \dots$

(E) , (E)

$$E = [- +] [1 - F(*)] - [-] F(*). \quad (1.14)$$

E / λ , (2004) , $(\sum_{i=1} \leq G)$, λ

Threshold 2:

$$G^{(2)} = \sum_{i=1} F^{(-1)} \left(\frac{-(\theta^- + 1) +}{- +} \right), \quad (1.1)$$

$$\theta^- = \frac{+ - (- +) F(0)}{- +} - 1. \quad (1.1')$$

C 1 $G^{(1)} \leq G^{(2)}$

$$F^*(*) = \frac{(- +)}{(- +)}. \quad (1.20)$$

C 2 $G^{(2)} \leq G < G^{(1)}$

$$F^*(*) = \frac{-(\theta + 1) +}{- +}. \quad (1.21)$$

C $G < G^{(2)}$

$$G_i = \sum_{i=1} F^{-1} \left(\frac{-(\theta + 1) +}{- +} \right) < G. \quad (1.22)$$

(2000)

1 ∇ t^{-0} \cdot t \cdot u \cdot v \cdot w \cdot x \cdot y \cdot z \cdot \dots

(1) ∇ t^{-0} \cdot t \cdot u \cdot v \cdot w \cdot x \cdot y \cdot z \cdot \dots

(2) ∇ t^{-0} \cdot t \cdot u \cdot v \cdot w \cdot x \cdot y \cdot z \cdot \dots

(3) ∇ t^{-0} \cdot t \cdot u \cdot v \cdot w \cdot x \cdot y \cdot z \cdot \dots

(2) ∇ t^{-0} \cdot t \cdot u \cdot v \cdot w \cdot x \cdot y \cdot z \cdot \dots

(200) ∇ t^{-0} \cdot t \cdot u \cdot v \cdot w \cdot x \cdot y \cdot z \cdot \dots

(*) ∇ t^{-0} \cdot t \cdot u \cdot v \cdot w \cdot x \cdot y \cdot z \cdot \dots

(**) ∇ t^{-0} \cdot t \cdot u \cdot v \cdot w \cdot x \cdot y \cdot z \cdot \dots

(2010) ∇ t^{-0} \cdot t \cdot u \cdot v \cdot w \cdot x \cdot y \cdot z \cdot \dots

(t) ∇ t^{-0} \cdot t \cdot u \cdot v \cdot w \cdot x \cdot y \cdot z \cdot \dots

$\int_0^1 F(x) dx = \sum_{n=1}^{\infty} \left[\frac{1}{n} \int_0^1 \frac{f(x)}{x^n} dx - \frac{1}{n} \int_0^1 \frac{f(x)}{x^{n+1}} dx \right]$

$\pi_2 = \sum_{n=1}^{\infty} \left[\frac{1}{n} \int_0^1 \frac{f(x)}{x^n} dx - \frac{1}{n} \int_0^1 \frac{f(x)}{x^{n+1}} dx \right]$

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$(E A)^t = \sum_{i=1}^n C^i(\dots) - \sum_{i=1}^n C^i(\dots)$

II (2006)

(1.24)

$$\begin{aligned}
 & \dots = \dots \neq \dots \geq \dots + 1 \dots + 2 \dots \\
 & \dots < \dots + 1 \dots + 2 \dots \\
 & \dots \geq \dots + 1 \dots + 2 \dots \\
 & \dots < \dots + 1 \dots + 2 \dots
 \end{aligned}$$

$$\dots \quad 1 \quad 0 \leq 1 \leq 1 \quad \dots \quad 1 = \dots + 1 \dots + 2 \dots$$

Handwritten musical notation on a staff. The notation includes various notes, rests, and symbols. A blue dot is placed above a note in the middle of the staff. A circled 'o' is located at the beginning of the staff. The text "(2011)" is written in blue ink above the staff. The notation is somewhat abstract and appears to be a form of musical shorthand or a specific notation system.

Handwritten musical notation on a staff. The notation includes various note values (quarter, eighth, and sixteenth notes), rests, and symbols such as 't', 'o', and 'f'. The notation is dense and appears to be a complex piece of music, possibly a score for a specific instrument or voice part. The notes are written in black ink on a white background.

The image displays a highly complex musical score, likely a score for a large ensemble or orchestra. The notation is dense and intricate, featuring a variety of note values, rests, and dynamic markings such as *f* (forte) and *ff* (fortissimo). The score is heavily annotated with black dots and lines, which appear to be editorial or performance markings. Several blue annotations are present, including the year "(2010)" in three locations and "(2011)" in one location. The overall appearance is that of a detailed and possibly revised musical manuscript.

The image displays a complex musical score consisting of approximately 12 staves of music. The notation is dense, featuring a variety of note values, rests, and dynamic markings. Several specific points in the score are highlighted with circled numbers: (1) is located on the lower left, (2) is in the middle left, (3) is on the lower left, (4) is in the middle right, (5) is on the lower left, and (6) is in the lower middle. The overall appearance is that of a detailed musical manuscript or score page.

The musical score on page 21 consists of a vocal line and piano accompaniment. The vocal line begins with a treble clef and a common time signature (C). It features a series of notes, including a half note followed by a quarter note, and a melodic phrase that repeats. The piano accompaniment is written in a grand staff (treble and bass clefs) with a common time signature. It includes complex rhythmic patterns, such as sixteenth and thirty-second notes, and rests. There are several dynamic markings, including *mf* (mezzo-forte) and *f* (forte). The score is divided into measures by vertical bar lines, and some measures contain repeat signs or first/second endings. The overall style is characteristic of late 19th or early 20th-century music.

1.3.1 One-Way Substitution

Let $\mathcal{A} = \{a_1, \dots, a_N\}$ and $\mathcal{B} = \{b_1, \dots, b_N\}$ be two alphabets of size N . A one-way substitution is a mapping $f: \mathcal{A} \rightarrow \mathcal{B}$ defined by a permutation π of $\{1, \dots, N\}$ such that $f(a_i) = b_{\pi(i)}$. The inverse mapping $f^{-1}: \mathcal{B} \rightarrow \mathcal{A}$ is defined by $f^{-1}(b_j) = a_{\pi^{-1}(j)}$.

Let $\mathbf{x} = (x_1, \dots, x_N)$ and $\mathbf{y} = (y_1, \dots, y_N)$ be two vectors in \mathcal{A}^N and \mathcal{B}^N respectively. The one-way substitution f maps \mathbf{x} to \mathbf{y} if and only if $y_i = f(x_i)$ for all $i \in \{1, \dots, N\}$. The probability of a vector \mathbf{x} is denoted by $P(\mathbf{x})$ and the probability of a vector \mathbf{y} is denoted by $P(\mathbf{y})$.

The probability of a vector \mathbf{y} is given by $P(\mathbf{y}) = \sum_{\mathbf{x} \in \mathcal{A}^N} P(\mathbf{x}) \delta(\mathbf{y} - f(\mathbf{x}))$, where $\delta(\mathbf{y} - f(\mathbf{x}))$ is the Dirac delta function. The probability of a vector \mathbf{x} is given by $P(\mathbf{x}) = \sum_{\mathbf{y} \in \mathcal{B}^N} P(\mathbf{y}) \delta(\mathbf{x} - f^{-1}(\mathbf{y}))$.

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$$P(\vec{y}) = - \sum_{i=1}^N \left(\dots \right) + \int_{\mathcal{A}^N} G(\vec{y}, \vec{x}) F(\vec{x}), \quad (1.31)$$

$$G(\vec{y}, \vec{x}) = \sum_{i=1}^N \sum_{j=1}^N \dots + \sum_{i=1}^N \dots - \sum_{i=1}^N \pi_i \dots$$

$$i + \sum_{j=1}^N \dots = \dots = 1, \dots, N, + \sum_{j=1}^N \dots = \dots = 1, \dots, N. \quad (1.32)$$

The image shows a complex musical score with multiple staves. The notation includes notes, rests, and various mathematical symbols. A prominent feature is the symbol $G(\rightarrow, \rightarrow)$ appearing on one of the staves. The score is dense and appears to be a technical or mathematical composition rather than a standard piece of music.

t • • • (2010

$$\begin{aligned}
 & + \int_0^1 \int_0^1 (1-x)(1-y) \dots \\
 & - \int_0^1 \int_0^1 (1-x)(1-y) \dots \\
 & + \int_0^1 \int_0^1 (1-x)(1-y) \dots \\
 & + \int_0^1 \int_0^1 (1-x)(1-y) \dots \\
 & - \int_0^1 \int_0^1 (1-x)(1-y) \dots \\
 & + \int_0^1 \int_0^1 (1-x)(1-y) \dots
 \end{aligned} \tag{1.35}$$

(2004)

$$G(x, y) = \int_0^1 \int_0^1 \dots$$

$$F_1(x) + \frac{(2+x)-1}{(1+x)-(2+x)} G(x, y) = \frac{(1+x)-1}{(1+x)-(2+x)}, \tag{1.36}$$

$$F_2(x) + \frac{(2+x)-1}{(2+x)-2} \left[G(x, y) - F(x, y) \right] = \frac{(2+x)-1}{(2+x)-2}. \tag{1.3}$$

$$\begin{aligned}
 & F(x) = \int_0^1 \int_0^1 \dots \\
 & G(x, y) = \int_0^1 \int_0^1 \dots \\
 & F_2(x) + G(x, y) - F(x, y) = \dots \\
 & \dots
 \end{aligned}$$

Table 1.1

\mathbb{K}	
β	
\mathbb{K}	
$0 < \alpha < 1$	
$G(\cdot)$	

1.3.2 Two-Way Substitution

$\sigma = \beta \sigma_1$
 $E \mathbb{K} C = \sigma G_1(\mathbb{K})$
 $G_1(\cdot) = \int_0^\infty (1 - e^{-\sigma t}) \dots$
 $\mathbb{K} = 0, \dots, N$
 $\mathbb{K}^* = 0, \dots, N$

$$E \mathbb{K} C(\mathbb{K}_1^*, \mathbb{K}_2^*, \dots, \mathbb{K}^*) = \frac{1}{2} N^2 + \sigma_1 \sum_{i=1}^N \beta_i \mathbb{K}_i^* \quad (1.3)$$

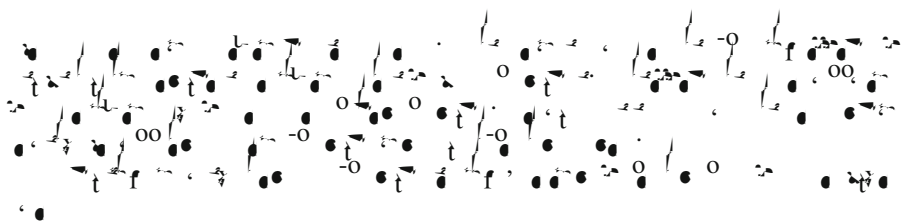
$\mathbb{K} = 1, \dots, N$
 $\mathbb{K}^* = 0, \dots, N$

1 $\sqrt{t} \begin{pmatrix} 1 \\ 0 \\ \vdots \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ \vdots \end{pmatrix} \dots$

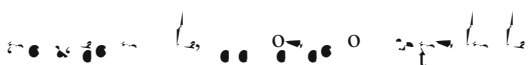
2

$\begin{pmatrix} 1 \\ 0 \\ \vdots \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ \vdots \end{pmatrix} \dots$

$E^{\infty} C(K)$



$$\begin{aligned}
 C & 1. \sum_{i=1}^2 [\dots + (\dots)] && \begin{cases} 1 \leq 1 & 2 \leq 2 \end{cases} \\
 C & 2. \dots + \dots (1-1, 2-2) && \begin{cases} 1 > 1 & 2 \leq 2 \end{cases} \\
 & + 2[(2-2)-(1-1)]^+ - 1[(1-1)-(2-2)]^+ \\
 C & \dots + \dots (2-2, 1-1) && \begin{cases} 1 \leq 1 & 2 > 2 \end{cases} \\
 & + 1[(1-1)-(2-2)]^+ - 2[(2-2)-(1-1)]^+ \\
 C & \dots - \dots (\dots) && \begin{cases} 1 > 1 & 2 > 2 \end{cases}
 \end{aligned}
 \tag{1.41}$$



$$\begin{aligned}
 E[\pi(1, 2)] &= E \left[\dots + 2 \dots [(1-1)^+] - 1 \dots - 2 \dots \right. \\
 & \quad \left. + 1 [1-1-(2-2)]^+ + 2(2-2)^+ - 1(1-1)^+ \right. \\
 & \quad \left. - 2[2-2-(1-1)]^+ \right] \\
 &= 1 \left[\int_0^1 \dots + \int_1^\infty \dots \right. \\
 & \quad \left. + \int_0^1 \int_2^\infty (2-2) \dots \right] \\
 &+ 2 \left[\int_0^2 \dots + \int_2^\infty \dots \right. \\
 & \quad \left. + \int_0^1 \int_2^\infty (1-1) \dots \right] \\
 &- 1 \dots + 1 \left[\int_0^1 \int_0^2 (1-1) \dots \right. \\
 & \quad \left. + \int_0^{1+2-2} \int_2^{1+2} (1+2-1-2) \dots \right] \\
 &+ 2 \left[\int_0^1 \int_0^2 (2-2) \dots \right]
 \end{aligned}$$

$$\begin{aligned}
 & + \int_0^{1+2^{-1}} \int_1^{1+2^{-2}} (1+2^{-1}-2^{-2}) (1, 2) \dots \\
 - 1 & \left[\int_1^\infty \int_2^\infty (1-1) (1, 2) \dots \right]
 \end{aligned}$$

t^{-0} ...

$F_4(t = \frac{1}{2}) \approx 1$, $3 = 1 + 1(2 - 2)$, $F_3(\frac{1}{3} = \frac{1}{3}) \approx 1$, $4 = 2 + 2(1 - 1)$

Musical notation including notes, rests, and stems across multiple staves.

1 $\int_{t_0}^{t_1} \dots$

$$\frac{\partial \mathcal{E}[\Pi]}{\partial \lambda_2} = -$$

$$\frac{1}{2}(\epsilon_1 + \epsilon_2) - \mu_1 = \frac{1}{2}(\epsilon_1 - \epsilon_2) + \mu_2$$

$$\mu_1 = \frac{\epsilon_1 + \epsilon_2}{2} - \mu_2 \tag{1.60}$$

$$\mu_2 = \frac{\epsilon_1 - \epsilon_2}{2} + \mu_1 \tag{1.61}$$

$$\epsilon_1 = \frac{1}{2}(\epsilon_1 + \epsilon_2) - \mu_1 + \frac{1}{2}(\epsilon_1 - \epsilon_2) + \mu_2 = \mu_1 + \mu_2$$

$$\begin{aligned}
 & - (\alpha + \beta - \gamma) \\
 & = (\alpha - \beta)(\alpha + \beta) - (\alpha - \gamma) + (\beta - \gamma)\mu \\
 & - (\alpha + \beta) \int_{\gamma}^{\infty} (\varepsilon - \gamma) f(\varepsilon) d\varepsilon, \tag{1.63}
 \end{aligned}$$

$$\begin{aligned}
 & + E[\Pi_2] = -(\alpha + \beta) \cdot \dots \quad \int_{\gamma}^{\infty} E[\Pi(\gamma_1, \gamma_2, \alpha, \beta)] = E[\Pi_1] \\
 & \frac{\partial E[\Pi]}{\partial \gamma} = (\alpha - \beta) + (\alpha - \beta) + (\alpha + \beta)[1 - F(\gamma)](1 - \alpha) + \\
 & - \{(\alpha - \beta) + (\alpha + \beta)[1 - F(\gamma)]\}. \tag{1.64}
 \end{aligned}$$

$$\frac{\partial^2 E[\Pi]}{\partial \gamma^2} = -(\alpha + \beta)(1 - \alpha)^2 f(\gamma) - (\alpha + \beta)^2 f(\gamma), \tag{1.65}$$

$$\begin{aligned}
 & \frac{\partial^2 E[\Pi]}{\partial \alpha \partial \beta} = -(\alpha - \beta + \gamma)(1 - \alpha) f(\gamma) - (\beta - \gamma)(1 - \beta) f(\gamma). \tag{1.66} \\
 & \dots = (\alpha - \beta) (\dots), \quad | \alpha | < 0, \quad | \beta | = \frac{1}{2} [(1 - \alpha)(1 - \beta) - \gamma^2] > 0,
 \end{aligned}$$

References

(200). ∇ t ∇ -o • t • u • • •

(3), 160 161.

(2005). ∇ t ∇ -o • t • u • • •

(2005). ∇ t ∇ -o • t • u • • •

(2004). ∇ t ∇ -o • t • u • • •

Musical score consisting of multiple staves. The notation includes notes, rests, and dynamic markings such as *f* and *E*. The score is annotated with various numbers and symbols:

- Staff 1: (1, 5), I (2), 3, 2, 40.
- Staff 2: (1, 5), I (2), 153, 162.
- Staff 3: (1, 6), E, (1), 2, 42.
- Staff 4: (1, 1), E, (2), 121, 12.
- Staff 5: (2001), (3), 334, 351.
- Staff 6: *NFO*, I (1), 4, 63.
- Staff 7: (2002), (4), 313, 335.
- Staff 8: (2000).

1 \sqrt{t}^{-0} ...

(2000). $E t$ (3), 213 22.

(2010). t^{-0} , 202(1), 10 113.

(2010). o oo t^{-0} 5 6 .

(200). t 46 . (.3 .163 (2)3! -)4((-22.1343 -1.1 , 3. (2